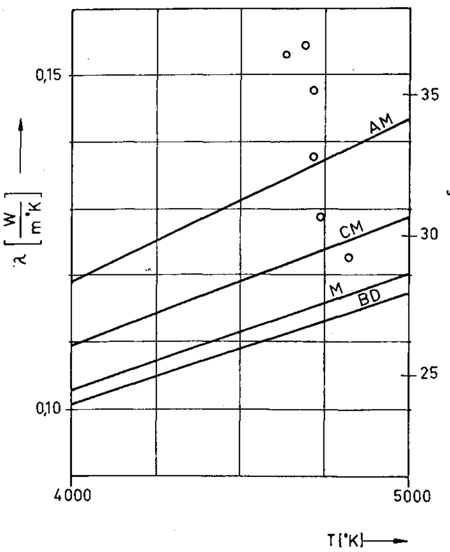

Fig. 4 Heat conductivity, directly measured and theoretical values; AM: Amdur, Mason,[9] CM: Collings, Menard,[2] M: Matula,[4] BD: Bunting, Devoto.[5]

wall had been obtained for several times after the reflection of the shock. The results of this evaluation are given in Fig. 3 which shows the temperature boundary layer at the end wall for four different times, i.e., $t_1 = 60\ \mu s$, $t_2 = 120\ \mu s$, $t_3 = 180\ \mu s$, and $t_4 = 300\ \mu s$ after the reflection of the shock; x represents the distance from the wall measured on the film.

From these temperature profiles the derivatives with respect to space and time are calculated. For this purpose the curves in Fig. 3 are approximated by polynomials which are adapted to the curves by a procedure involving an integration step in order to get smooth functions for differentiation. Using these functions the thermal conductivity λ is calculated from Eqs. (5–9) for several distances x from the end wall, representing simultaneously several slightly different temperatures. Results of this procedure for a series of experiments are given in Fig. 4 together with results obtained by former authors using the potential law $\lambda \sim T^{\sigma}$ with different values of σ.

An estimate shows that the maximum error is of the order of $\pm 20\%$. The main part of this error is due to the differentiation procedure involved in the numerical evaluation. The scattering of the experimental results has the same range as found with the potential law results mentioned. Therefore, in this way no decision can be made to support one of the cited measurements. The main aim of this Note is to present a new method to measure directly the thermal conductivity of gases. The first preliminary experimental results show that this method provides an accuracy at least comparable with the indirect measurements.

References

[1] Smiley, E. F., "The Measurement of the Thermal Conductivity of Gases at High Temperatures with a Shock Tube: Experimental Results in Argon at Temperatures between 1000°K and 3000°K, " Ph.D. thesis, 1957, Catholic University of America, Washington, D. C.

[2] Collins, D. J. and Menard, W. A., "Measurements of Thermal Conductivity of Noble Gases in the Temperature Range 1500 to 5000°K," *Journal of Heat Transfer*, Vol. C 88, No. 1, Feb. 1966, pp. 52–56.

[3] Camac, M. and Feinberg, R. M., "Thermal Conductivity of Argon at High Temperatures," *Journal of Fluid Mechanics*, Vol. 21, Pt. 4, April 1965, pp. 673–688.

[4] Matula, R. A., "High Temperature Thermal Conductivity of Rare Gases and Gas Mixtures," *Journal of Heat Transfer*, Vol. C 90, No. 3, Aug. 1968, pp. 319–327.

[5] Bunting, J. O. and Devoto, R. S., "Shock Tube Study of the Thermal Conductivity of Argon," SUDAAR 313, July 1967, Dept. of Aeronautics and Astronautics, Stanford University, Palo Alto, Calif.

[6] Smeets, G., "Bestimmung der Wärmeleitfähigkeit heisser Gase aus der Temperaturgrenzschicht im Stossrohr," *Zeitschrift fuer Naturforschung*, Vol. 20a, No. 5, May 1965, pp. 683–689.

[7] Hilsenrath, J., Messina, C. G., and Klein, M., "Table of Thermodynamic Properties and Chemical Composition of Argon in Chemical Equilibrium, Including Second Virial Corrections, from 2400°K to 35000°K," AEDC-TR-66-248, Dec. 1966, Arnold Engineering Development Center, Arnold Air Force Base, Tenn.

[8] Alfs, A., "Erzeugung einer kleinen Serie von partiellen Rubinlaser-Riesenpulsen," *Proceedings of the 8th International Congress on High-Speed Photography*, Wiley, New York, 1968, pp. 281–283.

[9] Amdur, I. and Mason, E. A., "Properties of Gases at Very High Temperatures," *The Physics of Fluids*, Vol. 1, No. 5, Sept.-Oct. 1958, pp. 370–383.

Earth Escape Regions near the Moon

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SPACECRAFT powered by microthrust may find it convenient to escape Earth by use of the moon's attraction. A method of achieving this which has previously been considered[1] is to orbit the spacecraft round the Earth in a direction contrary to that of the moon. A near miss of the moon is then just sufficient to attain escape velocity. A disadvantage of the contra-rotating orbit is that the Earth's axial spin makes it more difficult to place a spacecraft in its initial "near Earth" orbit.

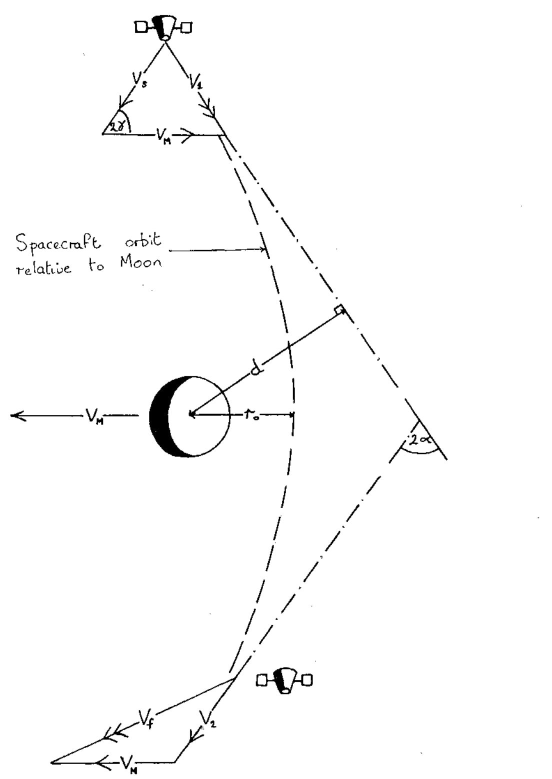

Fig. 1 Spacecraft orbit about moon.

Received May 26, 1970. The basis of this work was done while the author was at the College of Aeronautics, Cranfield, England.

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Initial orbits in planes which are skew to the moon's orbital plane, are more easily achieved. Expansion of these by microthrust tends to result in a nearly circular orbit at moon radius,[2] with the magnitude of the spacecraft's velocity close to that of the moon's. On passing behind the moon, the spacecraft can use the moon's attraction to increase its velocity, often to values well beyond Earth escape velocity.

Figure 1 shows the moon and spacecraft orbit, viewed along the Earth-moon line. It is assumed that while the spacecraft is in the vicinity of the moon, the Earth's attraction will act similarly on their respective motions and can be neglected for their relative motion. The initial velocity of the spacecraft relative to the moon ($\mathbf{V}_1$) is given by the spacecraft's velocity relative to Earth ($\mathbf{V}_s$) minus the moon's velocity ($\mathbf{V}_M$). As $\mathbf{V}_s$ and $\mathbf{V}_M$ are of equal magnitude and inclined at an angle, shown in Fig. 1 as 2γ, we have

$$V_1 = 2V_M \sin\gamma \quad (1)$$

The hyperbolic orbit of the spacecraft past the moon, results in a final velocity with respect to the moon (V_2), which is equal in magnitude to V_1, but inclined to V_1 at an angle shown in Fig. 1 as 2α. The final velocity of spacecraft with with respect to Earth ($\mathbf{V}_f$) is given by the resultant of $\mathbf{V}_2$ and $\mathbf{V}_M$, that is

$$V_f^2 = V_M^2 + V_2^2 - 2V_MV_2\cos[(\pi/2) + 2\alpha - \gamma] \quad (2)$$

Substituting for V_2 from Eq. (1), with $V_2 = V_1$, Eq. (2) can be written

$$(V_f^2 - V_M^2)/V_M^2 = 4\sin\gamma[\sin\gamma + \sin(2\alpha - \gamma)] \quad (3)$$

If suffix ( )0 refers to conditions of closest approach to the moon, and d is the distance from $\mathbf{V}_1$ to the moon center, (i.e., the miss distance for a nonattracting moon) then conservation of angular momentum gives

$$V_0 r_0 = dV_1 \quad (4)$$

conservation of energy gives

$$V_0^2 - 2\mu/r_0 = V_1^2 \quad (5)$$

where μ is the moon's gravitational constant, and the geometrical properties of a hyperbola give

$$d = r_0(\tan\alpha + \sec\alpha) \quad (6)$$

Thus, from Eqs. (1, 4, 5, and 6) α is related to γ by

$$\operatorname{cosec}\alpha = 1 + 4V_M^2(r_0/\mu)\sin^2\gamma \quad (7)$$

Maximum V_f is seen from Eq. (3) to require

$$2\alpha - \gamma = \pi/2 \quad (8)$$

corresponding to V_2 being aligned with the moon's velocity.

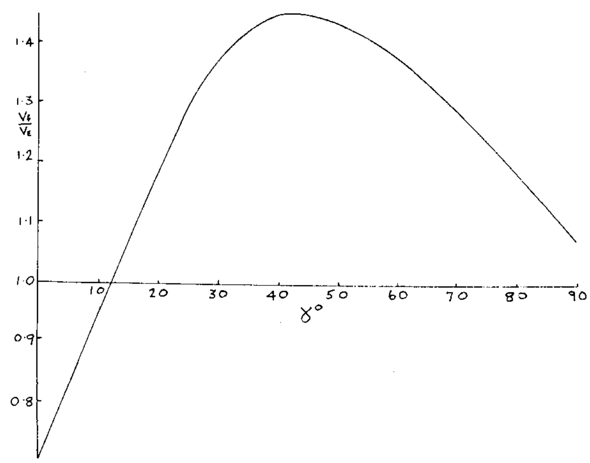

Fig. 2 Ratio of maximum velocity to escape velocity.

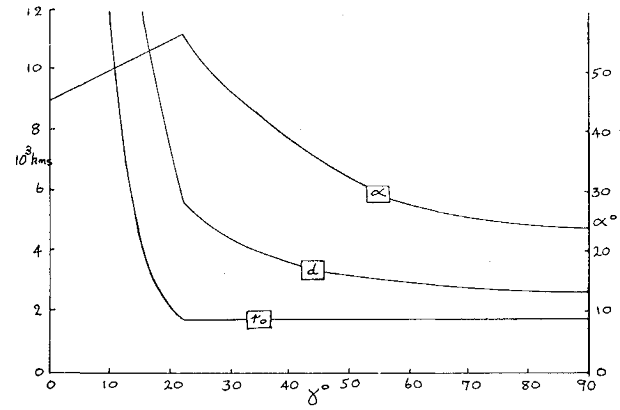

Fig. 3 Values of d, r_0, and α for the velocities achieved in Fig. 2.

Substituting Eq. (8) into Eq. (3), gives maximum V_f as

$$V_{f\max}/V_M = 1 + 2\sin\gamma \quad (9)$$

The Earth escape velocity at moon radius (V_E) is $(2)^{1/2}V_M \sim 1.44$ km/sec. Hence from Eq. (9), Earth escape is possible for $\gamma > 12°$, that is for a spacecraft orbital plane inclined to the moon's orbital plane at greater than 24°.

The moon miss distance can be obtained from Eqs. (7) and (8). For $\gamma > 22°$ it is found that r_0 is less than the moon surface radius (taken as 1750 km) and a collision results. To avoid this, for $\gamma > 22°$ α is obtained from Eq. (7) with r_0 equal to 1750 km.

In Fig. 2, the maximum velocity is compared with the Earth escape velocity; the associated values of r_0, d, and α are shown in Fig. 3. For a spacecraft orbit nearly perpendicular to that of the moon's (i.e., $\gamma \sim 45°$), up to 1.45 times the escape velocity can be achieved, giving a strongly hyperbolic Earth escape orbit.

At these larger velocities the direction of Earth escape may be important. This can be controlled by the moon approach used. Equation (3) applies for a spacecraft at moon radius; at other radii

$$(V_f^2 - V_M^2)/V_M^2 = 4\sin\gamma(\sin\gamma + \cos\gamma\sin2\alpha\cos\phi - \sin\gamma\cos2\alpha) \quad (10)$$

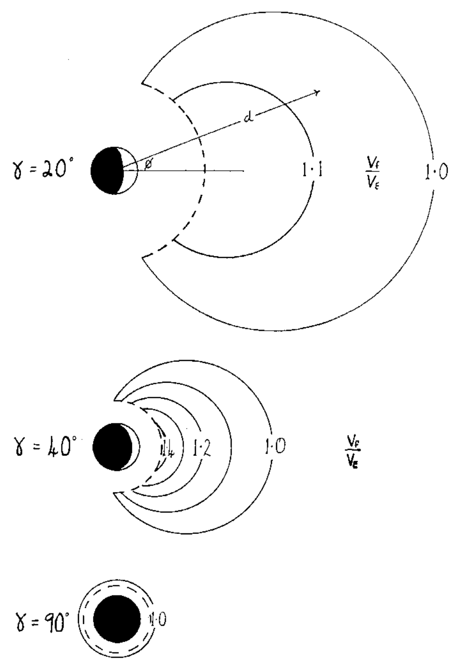

Fig. 4 Earth escape regions, near the moon.

where cosecϕ is the ratio of d to the difference in the moon and spacecraft radii (see Fig. 4).

From Eqs. (6, 7 and 10) it can be shown that contours of constant V_f in a (d, ϕ) coordinate system are circles, with center $(\mu \cot\gamma/(V_f^2 - V_M^2), 0)$ and radius

$$\mu[\{(V_f^2 - V_m^2)\tan\gamma\}^{-2} + \{2(V_f^2 - V_m^2)V_m^2\sin^2\gamma\}^{-1} + \{2V_m \sin\gamma\}^{-4}]^{1/2}$$

Examples of these circles for $\gamma = 20°$, $40°$ and $90°$ are shown in scale with the moon in Fig. 4. The "dashed" circular arcs intersecting the constant V_f circles, are the boundary of the moon impact region. Thus, the regions with V_f greater than a fixed value are crescent-shaped. The shading on the moon indicates its velocity with respect to the spacecaft. The moon's direction is indicated by it being shaded as if it were traveling away from the sun; and the fraction of disc shaded gives the speed as a fraction of $2V_M$.

For the high velocity producing orbits, the spacecraft has to be directed towards a small region just behind the moon with $\gamma \sim 40°$. For $\gamma \sim 20°$ the escape region (i.e., $V_f \geq V_E$) becomes large; in Fig. 4 it is shown to be a region of over 10,000 km in diameter.

The "head on" conditions when $\gamma = 90°$, permits good direction control, but the extent of the escape region is small.

References

[1] Hiller, H., "Use of the Moon's Attraction to Accelerate a Spacecraft to Hyperbolic Flight," TR 68203, 1968, Royal Aircraft Establishment.

[2] Burt, E. G. C., "On Space Manoeuvres with Continuous Thrust," *Planetary Space Sciences*, Vol. 15, Pergamon Press Ltd., London, 1967, pp. 103–122.

Stability of a Model Reference Control System

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Introduction

In a recent paper Lindh and Likins[1] compared the so-called infinite determinant method and a numerical implementation of Floquet theory for obtaining the regions of the parameter space corresponding to stability and instability of the null solution of a restricted class of linear, periodic coefficients, ordinary homogeneous differential equations. In this Note these methods will be applied to examine the stability of a model reference adaptive control system having sinusoidal input.

In recent years model reference adaptive control systems have proved very popular, particularly for practical applications to devices such as auto-pilots where rapid adaptation is required. The basic idea is shown in Fig. 1. The input $\theta_i(t)$ to the system is also fed to a reference model, the output of which is proportional to the desired response; the outputs of the model and system are then differenced to form an error

$$e(t) = \theta_m(t) - \theta_s(t) \quad (1)$$

Since the error is to be zero when the system is in the optimum state it is used as a demand signal for the adaptive loops which adjusts the variable parameters in the system to the desired value.

Various methods of synthesizing the adaptive loops have been proposed but the one that has proved most popular was that developed by Whittaker et al.[2] at the Massachusetts Institute of Technology and referred to as the "M.I.T." rule. Here the performance criterion is taken as the integral of error squared and a heuristic argument is given for reducing this over an unspecified period of time. This leads to a rule that a particular parameter K_i should be adjusted so that

$$\dot{K}_i = -Ge(\partial e/\partial K_i) \quad (2)$$

where G is the constant gain.

Although the "M.I.T." rule results in practically realizable systems, mathematical analysis of the adaptive loops, even for simple inputs, prove to be very difficult and it is usual in the design process to carry out much analogue computer simulation. The system equations are nonlinear and nonautonomous and since the nonlinearity is of the multiplicative kind, the mass of theory on instantaneous nonlinearities associated with the names of Lurẽ and Popov, in particular, is not applicable. In order to point out some of the difficulties we shall consider a simple first-order system having sinusoidal input.

Adaptive Control System

Since the intention, as previously mentioned, is to point out the difficulties involved in a stability investigation of a model reference adaptive control system, a simple first-order system with controllable gain will be considered.

Consider a model and system to be governed respectively by the equations

$$T\dot{\theta}_m(t) + \theta_m(t) = K\theta_i(t) \quad (3a)$$

$$T\dot{\theta}_s(t) + \theta_s(t) = K_vK_c\theta_i(t) \quad (3b)$$

where a dot denotes differentiation with respect to time t; the time constant T and model gain K are constant and known, but the process gain K_v is unknown and possibly time varying. The problem here is to determine a suitable adaptive loop to control K_c so that K_vK_c eventually equals the model gain K. The "M.I.T." rule gives

$$\dot{K}_c = -Ge(\partial e/\partial K_c) = Be\theta_m \quad (4)$$

where $B = GK_v/K$, and this leads to the scheme of Fig. 2.

If a sinusoidal input of magnitude $R \sin\omega t$ is applied at $t = 0$, when $\theta_m(t)$, $\theta_s(t)$ are zero and $K_vK_e \neq K$ and if subsequently K_v remains constant but K_c is adjusted according to Eq. (4), then using Eqs. (1, 3 and 4) the system equations become

$$T\dot{e}(t) + e(t) = (K - K_vK_c)R\sin\omega t \quad (5a)$$

$$\dot{K}_c = Be(t)\theta_m(t) \quad (5b)$$

Now consider that the adaption is switched on when the model response $\theta_m(t)$ has reached its steady-state value $\theta_{ms}(t)$ given by

$$\theta_{ms}(t) = [KR/(1 + T^2\omega^2)](\sin\omega t - T\omega\cos\omega t)$$

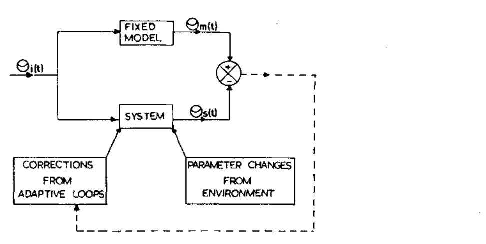

Fig. 1 Model reference adaptive control system.

Received July 21, 1970; revision received December 22, 1970. The author would like to express his gratitude to P. C. Parks of the Institute of Engineering Control, University of Warwick for his useful discussions concerning this problem. Thanks are due also to D. G. Evans and K. Underdown for carrying out the analogue computer simulation.

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